

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2

Branch: B.Tech (All)

Semester : 1

Date : 26/02/2020

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a)** nth derivative of $y = e^{-x}$ is
 (A) $-e^{-x}$ (B) $(-1)^n e^{-x}$ (C) $(-1)^{n+1} e^{-x}$ (D) none of these
- b)** If $y = \log(5-2x)$, then y_n equal to
 (A) $\frac{(-1)^n n!(-2)^n}{(5-2x)^{n+1}}$ (B) $\frac{(-1)^n n!(-2)^n}{(5-2x)^n}$ (C) $\frac{(-1)^{n-1} (n-1)!(-2)^n}{(5-2x)^n}$
 (D) $\frac{(-1)^n n!(-2)^n}{(5-2x)^{n-1}}$
- c)** If $y = \sin^{-1} x$, then x equal to
 (A) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (B) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$ (C) $1 - y + \frac{y^2}{2!} - \frac{y^3}{3!} + \dots$
 (D) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$
- d)** If $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$ then x equal to
 (A) $y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \dots$ (B) $y - \frac{1}{2!}y^2 + \frac{1}{3!}y^3 - \frac{1}{4!}y^4 + \dots$
 (C) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$ (D) none of these
- e)** $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{x} = \underline{\hspace{2cm}}$
 (A) 2 (B) $\log 2$ (C) $\log 15$ (D) $\log \left(\frac{5}{3} \right)$



- f) $\lim_{x \rightarrow \infty} x^n e^{-ax}$ (n being a positive integer and $a > 0$) = _____
 (A) -1 (B) 0 (C) 1 (D) None of these
- g) If $x = r \cos \theta$, $y = r \sin \theta$, then $\frac{\partial r}{\partial x}$ is equal to
 (A) $\sec \theta$ (B) $\sin \theta$ (C) $\cos \theta$ (D) $\operatorname{cosec} \theta$
- h) If $u = f\left(\frac{x}{y}\right)$ then
 (A) $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 0$ (B) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ (C) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u$
 (D) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$
- i) If $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)}$ is equal to
 (A) 1 (B) -1 (C) zero (D) none of these
- j) Conditions for $f(x, y)$ to be maximum are
 (A) $f_x = 0 = f_y$, $rt < s^2$, $r < 0$ (B) $f_x = 0 = f_y$, $rt > s^2$, $r < 0$
 (C) $f_x = 0 = f_y$, $rt > s^2$, $r > 0$ (D) $f_x = 0 = f_y$, $rt = s^2$, $r > 0$
- k) If $1, \alpha_1, \alpha_2, \dots, \alpha_{n-1}$ are the n roots of unity, then
 $(1 + \alpha_1)(1 + \alpha_2) \dots (1 + \alpha_{n-1})$ is equal to
 (A) $n-1$ (B) n (C) -1 (D) none of these
- l) If $y = \cos \theta + i \sin \theta$, then the value of $y + \frac{1}{y}$ is
 (A) $2 \cos \theta$ (B) $2 \sin \theta$ (C) $2 \operatorname{cosec} \theta$ (D) $2 \tan \theta$
- m) If every minor of order r of a matrix A is zero, then rank of A is
 (A) greater than r (B) equal to r (C) less than or equal to r
 (D) less than r
- n) An eigenvalue of a square matrix A is $\lambda = 0$. Then
 (A) $|A| \neq 0$ (B) A is symmetric (C) A is singular
 (D) A is skew-symmetric

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) If $y = \frac{x^4}{(x-1)(x-2)}$ then find y_n . (5)

b) Prove that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$ (5)

c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then show that (4)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}.$$

Q-3 Attempt all questions (14)



a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that (5)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0.$$

b) Expand $\tan^{-1} x$ up to the first four terms by Maclaurin's series. (5)

c) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ (4)

Q-4 Attempt all questions (14)

a) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{1-\cos x}$ (5)

b) Find the approximate value of $\sqrt{27}\sqrt[3]{1021}$ using partial differentiation. (5)

c) Expand $\log x$ in powers of $(x-2)$. (4)

Q-5 Attempt all questions (14)

a) If $u = \sec^{-1}\left(\frac{x^2 + y^2}{x-y}\right)$ then prove that (5)

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\cot u (\cot^2 u + 2).$$

b) Evaluate: $\lim_{x \rightarrow a} \log\left(2 - \frac{x}{a}\right) \cot(x-a)$ (5)

c) If $y = \cos x \cos 2x \cos 3x$ then find y_n . (4)

Q-6 Attempt all questions (14)

a) The power consumed in an electric resistor is given by $P = \frac{E^2}{R}$ (in (5)

watts). If $E = 200$ volts and $R = 8$ ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0,20 ohms?

b) Find the continued product of all the values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. (5)

c) Investigate for what values of λ and μ the equations (4)

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu,$$

have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

Q-7 Attempt all questions (14)

a) Find the eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$. (5)

b) Using De Moivre's theorem prove that (5)

$$\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$$

c) Prove that $\operatorname{sech}^{-1}(\sin\theta) = \log \cot \frac{\theta}{2}$. (4)

Q-8 Attempt all questions (14)



- a) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by Gauss-Jordan reduction method. (5)
- b) Find the fourth roots of unity and sketch them on the unit circle. (5)
- c) Examine for linear dependence of vectors (4)
(1, 2, -1, 0), (1, 3, 1, 2), (4, 2, 1, 0), (6, 1, 0, 1)
and find a relation between them if dependent.

